The Quaternion

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The Newsletter of the Department of Mathematics and Statistics

From Rogue Waves to Lump Waves

Rogue waves were once regarded as incredible as sea monsters and ghost ships; occasional reports of a ship that somehow survived a hundred-foot wave were put down to hyperbole and grog. After all, the standard statistical models predicted that such tall waves would be incredibly rare — especially out at sea. But then, on 1 January 1995, rogue wave hit the Draupner E platform off Norway:

It was 85 feet high.

Since then, satellites have confirmed that rogue waves are a common phenomenon, and indeed, Professor Nail Akhmediev of Australian National University estimates that at any given time, there are about ten rogue waves somewhere out there.

Outside of human curiosity, rogue waves pose a continuing threat to human life and property, so the theory needs to catch up with empirical science. We begin at the beginning. Oceanic rogue waves are surface gravity waves with wave heights much larger than expected (according to the standard Gaussian model); by surface gravity wave, we mean that it moves *on* the surface, unlike sound waves that move *through* water. Oceanographers used to think that wave heights would be normally distributed — which would mean that very tall rogue waves would be extremely rare.

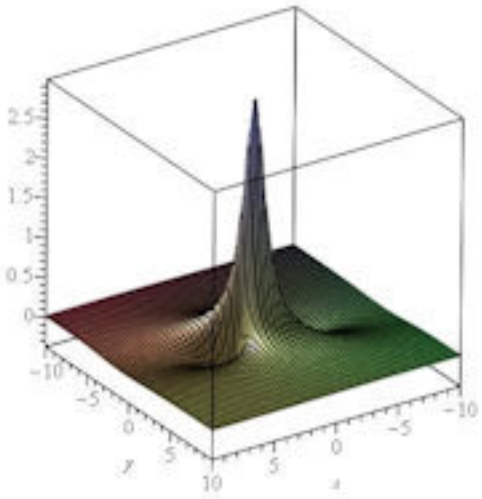
Waves were classically understood as manifestations of various wave equations, the most famous of which is

$$\frac{d^2}{dx^2} + x = 0,$$

whose solutions were the sine and cosine functions:

That is what waves looked like: sine or cosine waves. This wave equation is *linear* in that we only add terms; we don't multiply them.

Recently, mathematicians have been using *nonlinear* differential equations to model a rogue wave as a kind of *solitary wave* (or soliton). One of these, the Kadomtsev-Petviashvili (KP) equation, has a solution that looks like this "lump wave":



This lump wave and higher-order lump waves involve rational functions, i.e., quotients of polynomials. And the above lump wave is special: the denominator is a quadratic polynomial.

Now for a technical problem: if we have a solution to a differential equation, might it misbehave in ways that have nothing to do with rogue waves and everything to do with the model? (Mathematicians spend a lot of time on these technical issues because ... oceanographers and meteorologists like to have models that don't misbehave.) One collection of technical problems would be resolved if the denominator — which we remember is a quadratic polynomial - is always positive.

USF Professor Wen-Xiu Ma and USF alumnus Yuan Zhou determined when a quadratic function of several variables, expressed with a single vector variable, could be positive, as follows. The coefficients were matrices and vectors, like this: if the function f is of three variables x, y, z, then we can have a single vector variable v=[x,y,z] and a matrix A and two vectors $\mathbf{b}$ and $\mathbf{c}$ such that $f = v^T Av - 2\boldsymbol{b}^T v + \boldsymbol{c}$, where v^T is the matrix

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

They found conditions on A, $\mathbf{b}$, and $\mathbf{c}$ equivalent to f being positive everywhere.


Wen-Xiu Ma *Yuan Zhou*

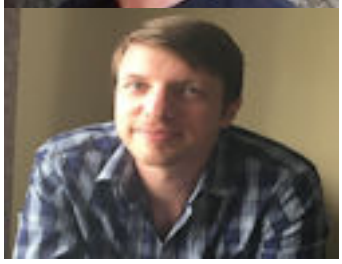
(Their paper is posted online.)

There is still a long way to go in testing these differential equations as descriptions of rogue waves. For example, suppose that a solution for a more sophisticated (and hopefully more accurate) model is a rational function whose denominator is not quadratic: we still do not have a good criterion for when non-quadratic polynomials are always positive.

Meanwhile, somewhere, out there, right now, ten rogue waves roam the seven seas...

Faculty News

Last Fall we welcomed two new faculty members:



Giacomo Micheli received his Ph.D. from the University of Zurich in 2015 and undergraduate degrees from University of Rome. Then he was a Postdoctoral Fellow at Massachusetts Institute of Technology, a Research Fellow at University of Oxford, and a Scientist at the École Polytechnique Fédérale de Lausanne. He works in algebraic number theory and cryptography and coding theory.

Joel A. Rosenfeld received his Ph.D. from the University of Florida in 2013 in Functional Analysis and Operator Theory. Prior to coming to USF, he had had a number of positions in interdisciplinary fields: from 2013 to 2018, he worked as a postdoctoral researcher in engineering departments, including Mechanical Engineering at the University of Florida, studying numerical methods in optimal control theory and fractional calculus. Subsequently, he joined the Department of Electrical Engineering and Computer Science at Vanderbilt University and held a position as a Senior Research Scientist Engineer in that same department.

In other news:

Jean-François Biasse received an NSF CAREER award of $450,000 over 5 years to work on the search for short vectors in ideal lattices. This mathematical problem is important to post-quantum cryptography because its hardness would guarantee the security of the most promising proposals for quantum resistant public-key schemes. This project investigates the potential weaknesses induced by the algebraic structure surrounding ideal lattices. This award also contains a strong outreach component, with the support for a Cybersecurity Summer camp for K-12 students in partnership with the USF Whitehatter Computer Security Club.

He also received an FC^{2}2 Capacity Building award of $75,000 over 1 year to work in collaboration with Rainer Steinwandt (FAU) on the design of online modules for emerging topics in cryptography. The proposed modules consist of three online mini-courses (1-2 credits each) on Blockchain technology, Post-Quantum Cryptography, and Quantum Cryptography. The award will also support events to promote the participation of under-represented groups in the field of cryptography, with a particular emphasis on the engagement of female students.

Dima Khavinson won the 2019 Outstanding Associate Editor Award, Journal of Mathematical Analysis and Applications. He also published a book: "Linear Holomorphic Partial Differential Equations and Classical Potential Theory" (co-authored with USF alumnus Erik Lundberg of Florida Atlantic University), published by the American Mathematical Society. He also coedited a book, "Analysis of Operators on Function Spaces, The Serguei Shimorin Memorial Volume", (with Alexandru Aleman of Lund University, Håken Hedenmalm of the Royal Institute of Technology at Stockholm, and Mihai Putinar of UC Santa Barbara), published by Birkhauser.

Dima Khavinson is the editor-in-chief and **Razvan Teodorescu** is the managing editor of the Springer journal *Analysis and Mathematical Physics*, which reached the top 7% of all mathematical journals ranked by Clarivate Analytics, with an impact factor of 1.792. The impact factor has more than tripled during their tenure, starting three years ago.

Razvan Teodorescu won the USF Jerome Krivanek Distinguished Teacher Award; the formal ceremony for this award was held on November 22. From right to left, USF President **Steven Currall**, Professor Teodorescu, and USF Provost **Ralph Wilcox**.

Wen-Xiu Ma was selected for the Clarivate Analytics list of Highly Cited Researchers for the 5th consecutive year (2015, 2016, 2017, 2018 and 2019). Highly Cited Researchers identifies the world's most influential contemporary researchers across 21 scientific fields. Over three thousand researchers earned the distinction by writing the greatest numbers of reports officially designated by Essential Science Indicators (ESI) as Highly Cited Papers — ranking among the top 1% most cited for their subject field and year of publication, earning them the mark of exceptional impact.

Student News

Undergraduate Math Clubs News

The USF Student Chapter of the Mathematical Association of America (MAA) meets every other week during the academic year, and features invited faculty and students doing interesting math presentations at the undergraduate level. This 2019 fall semester the math club is sponsoring a team of USF math undergraduates for participation in the prestigious Putnam Competition in December. Good luck to them!

The MAA math club also co-hosts the annual Math Picnic every fall semester at the USF Riverfront Park.

The officers are Alexander Mercier (President), and Keller Blackwell (Vice President).

USF Students Will Soar to Cross the Karman Line

One of the vibrant and active student organization at USF in Tampa, is the *Society of Aeronautics and Rocketry* (SOAR). The genesis of SOAR goes back to 2010. At the time, USF had a "Science Club" with **Matthew Chrzanowski** as president. Professor **Manoug Manougian** was asked to serve as faculty advisor. The Science Club was renamed the Society of Aeronautics and Rocketry (SOAR), and the rockets were labelled *Bull-istic*, a name stemming from the university's mascot, a bull.

SOAR team and Bull-istic

The goal is to perfect the launching of rockets into space to put satellites into orbit for scientific studies. With funds from NASA and the Student Government, SOAR proceeded to design, build rockets, and participate in competitions sponsored by NASA and the Florida Space Grant Consortium (FSGC). It didn't take long for SOAR to succeed and win awards.

At USF, SOAR members use the program to learn more than just launching rockets. Participants learn about the interconnectedness of the various STEM disciplines, in a cooperative environment, while engaged in science and engineering projects. With that background, and with encouragement from the Provost, **Ralph Wilcox**, and the Dean of Arts & Sciences, **Eric Eisenberg**, SOAR has been approved to participate in a competition, the first of its kind, hosted by *HeroX* and the *Base 11 Space Challenge*.

During the Base 11 Space Challenge, university students in the U.S. and Canada will design, build, and launch a one-stage liquid-propelled rocket. The university whose rocket reaches 63 miles above sea level (the Karman line) first, will receive the grand prize of $1 million dollars. Additional prizes and accolades are also awarded. The competition will take place in New Mexico. The devotion and passion shown by this group of students gives USF's SOAR program an edge to come back to USF with the grand prize.

Board members of SOAR

Finally, to students everywhere, Manougian says: Follow your dreams, and heed the words of Albert Einstein: "Imagination is more important than knowledge". That's how creativity is nurtured and innovations created.

Fifteen USF Students Inducted into Pi Mu Epsilon Honor Society

On April 19, 2019, the Florida Epsilon Chapter of Pi Mu Epsilon inducted the following 15 USF students as new members of the national mathematics honor society: Jaeden Ayala, Patrick Collard, Brandon Davey, Reagan Davey, Hamza Elhamdadi, Elizabeth Funk, Sean Hays, Alexander Mercier, Viviana Milla Angeles, Deanna Ramnarine, Alexa Scott, Thrisna Singh, Thomas Veith, Zachary Withers, and Xiankui Yang.