The Quaternion

The Newsletter of the Department of Mathematics and Statistics

Volume 23: Number 1; Fall, 2008

In This Issue

Why are the atomic clocks running fast?

Carol Williams *retired last December after forty years at USF, although she continues to be an active astronomer. She very kindly wrote this article on her recent research, assisted by Dima Khavinson.*

In 1695 Edmund Halley, using Newton's theory of gravity to describe the motions of the Earth and the Moon, tried to validate ancient records of solar eclipses. He found that Newton's formulas consistently gave values for the Moon's longitudes that were larger than those recorded for the eclipses. This problem was never solved by Newton or Halley. In 1786, Laplace accounted for about half of the discrepancy as being due to the gravitational forces of the other planets (besides Earth) on the Moon's orbit. The problem remained unsolved until 1939 when Spencer Jones found proportional discrepancies in the longitudes of the Sun, Moon, Mercury and Venus. It was he who finally persuaded astronomers that something, probably tidal friction, was slowing down the rotation of the Earth, causing a slow decline in the rate of mean solar time. The equations were not wrong; the time was!

This is how Spencer Jones' model works. Let's continue to measure days as the period of time from, say, midnight to midnight. Then, as the Earth's rotation is slowing down, the days get longer. Even if orbits do not change, the measurement of orbital periods (as a certain number of days) would give smaller values now than before. To calculate a longitude, one takes the time of an observation and uses this numerical value in the equation for the longitude. If we insert current values for the time into equations that still contain older values of the orbital periods, the calculated longitudes will be larger than their currently observed values.

This reasoning led astronomers to define a new timescale and invent a method of measuring it, with both the scale and the method being independent of the Earth's rotation. The timescale was called Ephemeris Time and it was to be measured by freezing the values of the orbital periods to the values they had at the start of 1900 and making special clocks that ticked at a rate that always gave this orbital period every time the planet finished one orbit. Atomic clocks were invented for this purpose and always give seconds that are of the same length. The length of an atomic second was calibrated using the time values from the equations for the orbit of the Moon. As an added bonus, the clocks give time very precisely, up to a billionth of a second. When measured with atomic clocks, the orbital periods have the same values as in 1900. If one inserts the atomic time of an observation into the equations, the calculated longitudes match observations. But atomic time and mean solar time do not give the same clock readings; atomic clocks run faster. To keep atomic time within 0.9 second of mean solar time, astronomers occasionally add a leap second to the atomic clock. So the number of seconds one counts in some years will be one second larger in atomic time than in mean solar time.

One would think that the story was finished, and to many it is. But some are not convinced that the Earth is slowing down by that much. Geophysicists have measured the deceleration of the Earth's rotation due to tidal friction and get a value that is about 100 times smaller than what the number of leap seconds suggests. Looking into this problem, Dr. Steven D. Deines, Engineer at Rockwell Collins in Cedar Rapids, Iowa, and I determined that time dilation, described by Einstein in his theories of relativity, sheds light on this problem. In particular, Einstein tells us that if one reference frame is moving with respect to another, identical clocks in the two frames will not run at the same rate.

All equations used to calculate the longitudes were established from Newton's theory of gravity before Einstein's theory of General Relativity was published. Thus no changes were ever incorporated into the equations to account for time dilation. When we calculate and observe the longitudes we are using two different reference frames that are moving with respect to one another. The equations for the calculations are established using a theoretical reference frame that is inertial; it is not rotating and its origin is located at the barycenter of the solar system. When we observe the longitudes from the Earth, we record a time of observation using mean solar time, measured on the rotating, orbiting Earth. A clock on the Earth cannot be running at the same rate as a clock sitting at the barycenter of the Solar System. Since atomic time was calibrated to match Ephemeris Time, it gives time at the barycenter. (Most timekeeping experts agree that this is the case although small corrections are added to insure this.)

Deines and I have determined the amount of time dilation needed to explain the discrepancy between mean solar time (proper time on the Earth) and atomic time (coordinate time of the equations). The lengths of seconds in the two timescales are different; one second of atomic time is shorter than one second of mean solar time. Atomic time will always run faster than mean solar time; leap seconds will always be necessary. Tidal friction can have the value determined by Geophysics but it will not change our result. Our calculations match the observed runoff between the two clocks up to 0.2%.

Yet some of the best scientists over the past several hundred years have supported the rotational deceleration hypothesis. Our calculations rise against hundreds of years of tradition. Naturally, we find ourselves in the middle of controversy. So be it. I am sure we are correct.

Knotting Math & Art at USF

Last fall, the R. Kent Nagle Lecture was the keynote speech for a multidisciplinary conference on **Knotting Mathematics and Art: Topology and Mathematical Art**, which was held at USF on Nov. 1 – 4. The Nagle Lecturer was John Horton Conway, who talked about finite objects, friezes, and tilings in *From Topology to Symmetry*. Subsequently, participants attended several parallel sessions of presentations by artists, mathematicians, and sometimes both (together in the same room!), along with assorted third parties – most notably chemists interested in the geometry and topology of nanostructures.

R. Kent Nagle Lecture

John Horton Conway is one of the world's most celebrated and prolific mathematicians. His contributions range through finite group theory, knot theory, number theory, and coding theory. He may be best known to the public for developing the *Game of Life* automaton and for his work on combinatorial game theory. A Fellow of the Royal Society, he has received the Junior Berwick Prize, the George Polya Prize of the London Mathematical Society, the Frederic Esser Nemmers Prize in Mathematics, and the Steel Prize of the American Mathematical Society.

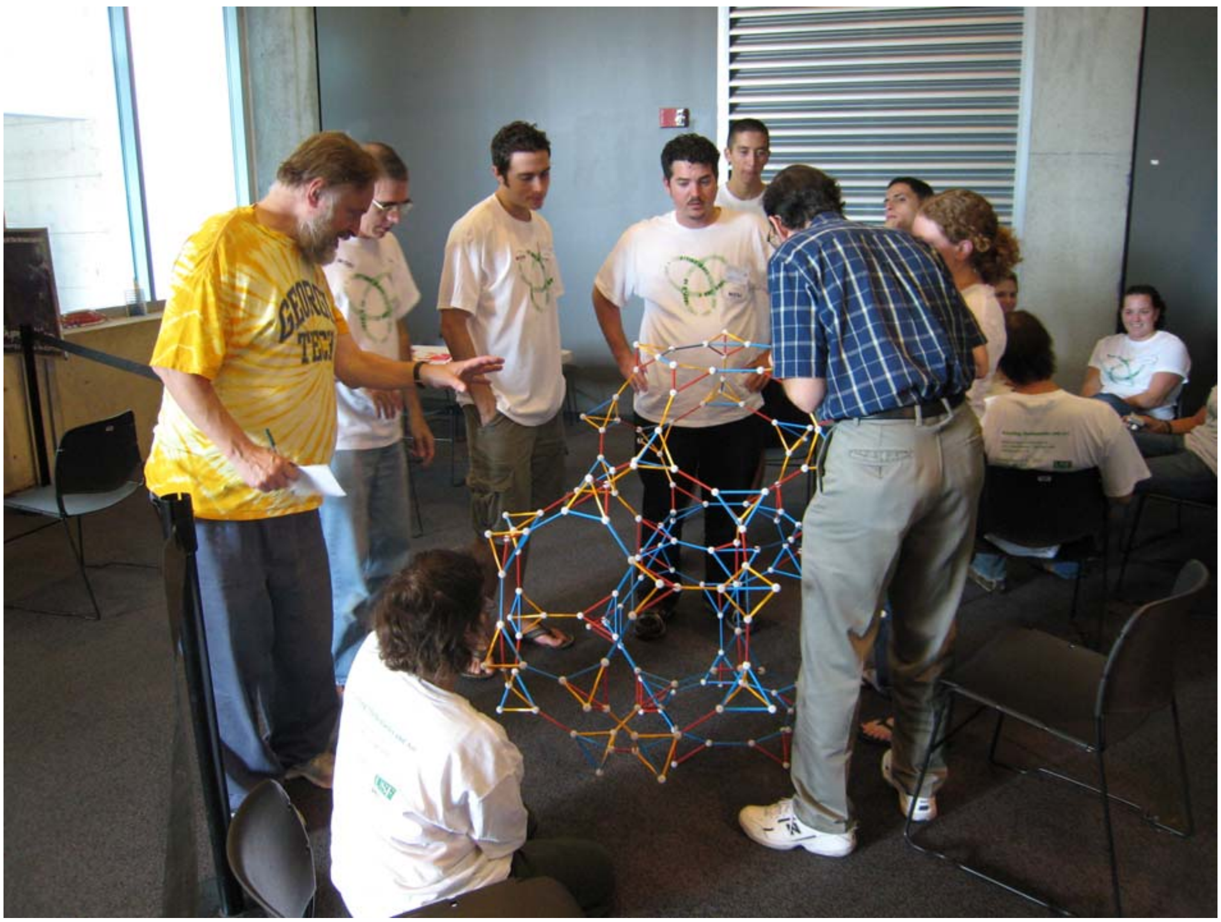
He outlined a new system for enumerating the fourteen (classes of) symmetry groups of the sphere, the seven (classes of) symmetry groups for friezes, and the seventeen (classes of) symmetry groups for tilings of the plane.

Low Dimensional Topology

The conference featured Thomas Banchoff of Brown University, who talked about *The Fourth Dimension and Salvador Dali*, J. Scott Carter of the University of South Alabama and artist Tony Robbin, who discussed their *Collaboration across the two cultures*. Similarly, artist Brent Collins and Carlo Séquin of the University of California, Berkeley, joined in their *Analysis and synthesis of intuitively conceived geometric art*. Charles Frohman of the University of Iowa talked about *The growth rate of the quantum hyperbolic invariants of Baseilhac and Benedetti*. Seiichi Kamada of Hiroshima University spoke *On braid description of surface-links in 4-space*. Mihail Khovanov of Columbia University described *Categorification in examples*. Yasutaka Nakanishi of Kobe University discussed *Local moves and Gordian complexes*. And author Ivars Peterson described *A Journey into Mathematical Art*. At the tail end of the conference, Conway talked about topology and geometry in three dimensions. These plenary addresses were accompanied by forty-nine presentations in parallel sessions.

Mathematical Art

Three exhibitions, collectively entitled *Rhythm of Structure: Beyond the Mathematics,* displayed contemporary work in mathematical art. The work of Brent Collins, Helaman Ferguson, Mike Fields, Bathsheba Grossman, George Hart, Charles Perry, Tony Robbin, and John Sims was featured in the Centre Gallery while the work of George Hart, Thomas Banchoff. Carlo Séquin, Chaim Strauss-Goodman were featured in the Museum of Science and Industry. And the work of Sol Lewit, Paulus Gerdes, and John Sims was featured at the Oliver Gallery. Students in the mathematics clubs (the USF chapters of the Mathematical Association of America and Pi Mu Epsilon) constructed a model of the parallel projection of a truncated four-dimensional 120-cell, with some kibbitzing by Conway (in yellow t-shirt at left), George Hart (standing left of Conway, with glasses), and Alex Feingold (in blue plaid shirt with back towards the camera).

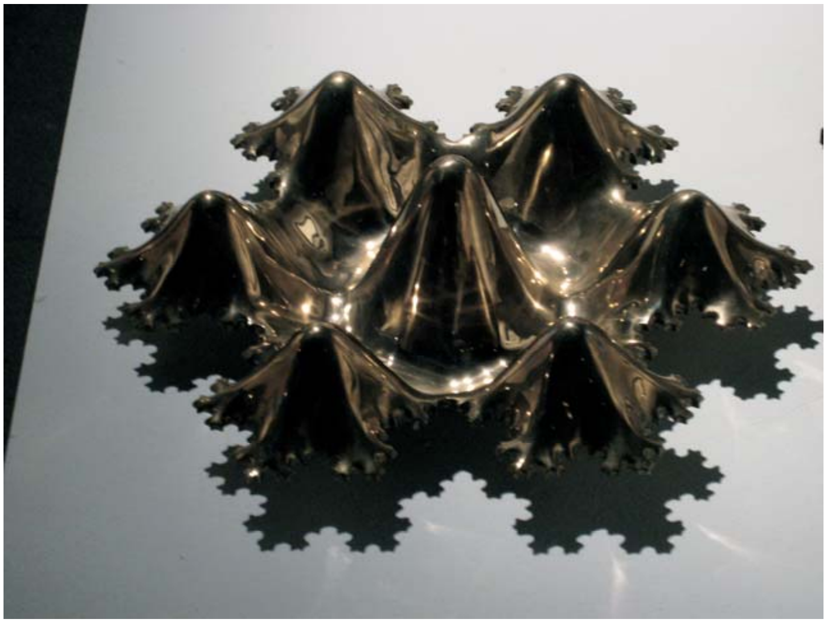


The resulting ball was displayed in the Museum of Science and Industry lobby ⇒⇒⇒⇒⇒⇒⇒⇒

This photo – somewhat adjusted from the original – is courtesy of George Hart, who has more posted at http://www.georgehart.com.

Art displayed ranged from paintings to sculptures like:

Dirichlet Laplace Snowflake, a bronze sculpture by Helaman Ferguson



For more information, the interested reader can consult the review article, ***Knotting Mathematics and Art****, by George Hart & Natasha Jonoska, in the* ***Journal of Mathematics and the Arts****, 2:1 (2008), pp. 47 - 51.*

Transitions

Carol Willliams retired last winter after forty years at USF. She received her B.A. from Connecticut College in 1962, during which time she worked with Heinz Eichhorn at Wesleyan College as a computer programmer for NASA (back when computers read their programs from punched computer tape). "I made DO-loops by pasting the tape into loops before feeding them through the reader."

She went to Yale in the post-Sputnik era, receiving her doctorate in astronomy at Yale in 1967. In that Apollo era, she began working on one of NASA's great fixations, the precise location of the Moon. When Eichhorn came to USF, she followed, and stayed after the rest of the USF astronomers departed in 1979. She works in celestial mechanics and astrometry, and has received an Outstanding Research Award from Sigma Xi and an Outstanding Teacher Award from USF.

She is now a professor emeritus, and as can be seen from the feature article, still very much interested in the precise location of the Moon.

Sherwin Kouchekian came to USF from the University of South Alabama, Mobile, where he was an Assistant Professor. He received a M.S. in mathematics from the Royal Institute of Technology, Stockholm (Sweden) in 1993 and a Ph.D. in mathematics from the University of Tennessee, Knoxville, in 2000. He spent two years as a postdoc at Virginia Tech and another year at the University of Kentucky, Louisville. He received an NSF grant in 2005, and works in operator theory, function theory, and mathematical physics.

Wonkuk Kim came to USF from SUNY Stony Brook, where he was a graduate student and received a Ph.D. in applied mathematics and statistics in 2007. He had received a B.S. in nuclear engineering (1989) and a M.S. in mathematics (1993) from Seoul National University (Korea). He works in statistical genetics, mixture models, survival analysis, and data mining.

Gangaram Ladde came to USF from the University of Texas at Arlington, where he was a Professor of Mathematics. He received his B.Sc. (First Class) in chemistry, mathematics and physics, from People's College, Nanded (India) in 1963, M.Sc. (First Class First) in mathematics from Marathwada University, Aurangabad (India) in 1965, and Ph.D. in mathematics from the University of Rhode Island in 1972. Dr. Ladde's research interests are in dynamic reliability analysis and control; stochastic modeling of dynamical processes in biological, chemical, engineering, medical, physical and social sciences; time series analysis and applications; deterministic and stochastic qualitative and quantitative properties of dynamic systems; stability theory; stochastic estimation and filtering; deterministic and stochastic control and differential games; multivariate/large-scale systems analysis; hereditary systems; stochastic modeling of network dynamics; multi-agent and multi-market/finance; stochastic approximation and statistical analysis; stochastic hybrid dynamical and extreme statistical analysis. Dr. Ladde has received several research awards and grants, and is the Founder and Editor in Chief of the journal *Statistical Analysis and Applications*, and serves on several other editorial boards as well.

Faculty News

Dimitry Khavinson and University of Northern Iowa Professor Genevra Neumann have answered an open question in astrophysics using complex analysis. Imagine that between the Earth and some very bright light source (like a quasar) there are n massive objects, small in size and relatively close to each other so that we could treat them as point masses lying on a common plane. Since the gravity of these massive objects bend the light from the light source, Earthlings perceive several images of the light source. If we can count the number of images of that light source, can we estimate the number of massive objects there are?

The "lens equation" that relates the images to the original source and to the massive objects involves a rational harmonic function of the form $r(z) = p(z)/q(z) - z^*$, where p and q are (complex) polynomials and "*" denotes complex conjugacy. The zeros of the function correspond to images of the light source. Khavinson and Neumann demonstrated that if $n > 1$ is the maximum of the degrees of p, q, then the number of zeros of r is at most $5n - 5$; from this it follows that if there are $n > 1$ massive objects, then there are at most $5n - 5$ images. This confirms a conjecture of University of Notre Dame Astronomy Professor Sun Hong Rhie, who had already proven that this upper bound can be achieved.

An article on this work appeared in the June/July issue of the *Notices of the AMS*, entitled *From the Fundamental Theorem of Algebra to Astrophysics: A "Harmonious "Path*, and is posted on-line at http://www.ams.org/notices/200806/tx080600666p.pdf.

Student Clubs

Last year the **Math Club** officers were: President Dane Harmon, Vice-president Jessica Couvertier, and Treasurer Ryan Grotheer. Fourteen regular club meetings were held every other Friday, with students, faculty and guests speaking on topics ranging from *The Mathematics of Web Programming and Design*, to *Secrets of Mental Math*, and *Careers in Actuarial Science*.

The Club participated in the conference **Knotting Mathematics and Art**, organized by the Department in November, helping mathematician George Hart build a model of the mathematical structure called "The Truncated 120 Cell" in the lobby of the Museum of Science and Industry (MOSI); see pictures at http://knotart.cas.usf.edu/graphicSite/. Math club members also attended the Suncoast Regional Meeting of the MAA in December at Eckerd College, and the Annual Meeting of the MAA Florida Section at Florida Southern College.

The USF Math Club has a new refurbished look thanks to Dane Harmon. Check them out at http://shell.cas.usf.edu/~mathclub/.

Last year, the officers of the Florida Epsilon Chapter of **Pi Mu Epsilon** were President Zach Jett, and Vice-president Joy D'Andrea. Our PME Chapter at USF inducted sixteen members in April: Eleonora Antoniou, Helen Barclay, Fernando Burgos, Jonathan Burns, Tyson DiLorenzo, Gorka Duralde, Dewey Estep, Lauren Fertig, Kenneth Kane, Daria Karpenko, Elena Rodriguez, Jason Rosendale (Math), Alison Sibol, Armando Signorini, Carolyn Silcott, and Brian Vohaska. Egor Dolzhenko, and Gabriel Zayas-Cabán won the PME Outstanding Scholar Award, and plaques were presented to them during the 2008 PME Induction Banquet in April, when the banquet's featured speaker was Dr. Patrick McDonald of New College of Florida, who spoke on "Emerging Science and Problems for the Young at Heart".

Student News

USF graduated 58 bachelor students this year: Jasper Adams, Louis Anderson, Bridget Asplund, *magna cum laude*, Krystal Baird, Helen Barclay, Michael Beidler, Stephen Best, Sandra Bird, Christina Bolanos, Gail Bostrom, Eric Bridges, Joel Brown, Andrew Burruss, Joy D'Andrea, Carissa Deneca, Sarah Dickmann, *summa cum laude*, Egor Dolzhenko, *magna cum laude*, Dewey Estep, Steven Farley, *summa cum laude*, Luke Gittens, Philip Grablow, *cum laude*, Ana Greathouse, Susanne Helms, Roger Hudson, Elliott Jenkins, Laurie Jones, Dahomey Kadera, *summa cum laude*, Dionesia Kalos, *summa cum laude*, Jonathan Kessack, Brent Lewis, Cody Ligon, Cheryl Little, Adrienne Lobascio, Carissa Lyons, Joan Marius, Keith McLaughlin, *summa cum laude*, Karen Michalski, Michael Nachtigal, *summa cum laude*, Layna Nolan, Maya Ozek, Lai Price, Damione Puopolo, Nicholas Reithmaier, Jason Rosendale, *summa cum laude*, Toby Skaria, Laurie Sosa, Elizabeth Sweet, Maya Tarleva, Jacqueline Taxdal, *summa cum laude*, Christopher Valdez, Brian Vohaska, Howard Vorder-Bruegge, *magna cum laude*, Misty Vorder-Bruegge, *cum laude*, Matthew Wanson, Mathew Williamson, *cum laude*, Trystal Woods, and Gabriel Zayas-Caban, *magna cum laude*.

We also graduated fourteen masters' students this year: Julie Cholet, Egor Dolzhenko (*Transducer Dynamics* under N. Jonoska), Gary Dowd, Christine Fitch, Diego Grilli, Vindya Kumari, Xi Liu, Jinghan Meng, Tilahun Muche, Ricardo Restrepo, Toby Tiller, Vien Truong, Misty Vorder-Bruegge, and Mathew Williamson (*Kauffman-Harary Conjecture for Virtual Knots* under M. Saito).

And we awarded twelve doctorates of philosophy to: Dhruba Adhikari (*Applications of Degree Theories to Non-linear Operator Equations in Banach Spaces* under A. Kartsatos), Irena Andreevska (*Mathematical Modeling and Analysis of Options with Jump-Diffusion Volatility* under Y. You), Ibrahimou Boubakari (*The Leray-Schauder Approach for the Topological Degree of Perturbed Maximal Monotone Operators* under A. Kartsatos), Ibtisam Daqqa (*Subconstituent Algebras of Latin Squares* under B. Curtin), John Davis III (*Identification of Parameters When the Density of the Minimum is Given* under A. Mukherjea), Daniela Genova (*Forbidding and Enforcing of Formal Languages, Graphs, and Partially Ordered Sets* under N. Jonoska), Florence George (*Johnson's System of Distributions and Microarray Data Analysis* under K. Ramachandran), Armando Hoare (*Parametric, Non-Parametric and Statistical Modeling of Stony Coral Reef Data* under C. Tsokos), Alfred Mbah (*On the Theory of Records and Applications* under C. Tsokos), Shou Hsing Shih (*Forecasting Models for Economic and Environmental Data* under C. Tsokos), Ana Staninska (*A Theoretical Model for Flexible Tiles Self-Assembly* under N. Jonoska & G. McColm), and Rodney Taylor (*Lagrange Interpolation on Leja Points* under V. Totik).