The Quaternion

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Feature Article: Approximations at USF

by Vilmos Totik

V. I Arnold, one of the leading mathematicians of our time, flatly defines mathematics as part of physics. According to this anything in mathematics that is irrelevant to the outside world is useless and avoidable.

But mathematics has its own ways of expanding. Internal coherence, easthetics, and the desire for completeness, coupled with curiosity and ingenuity of the researchers are the main driving forces behind new concepts and theories. It is a miracle that very often the expansions and developments arising from internal forces within mathematics are just what the outside world needs.

Probably the most typical example is the invention of complex numbers: they emerged from the desire to solve equations, and they have become the basis of powerful theories essential to modern physics and without which developments like electronics would not exist. A second example is the theory of matrices, which were developed in mathematics to help understand the geometry of space around us. (A matrix is a rectangular array of numbers.) Physicists noticed that some of Heisenberg's computations resembled matrix calculations, so they tried replacing the position and momentum variables with matrices. The resulting theory became so transparent that they were scared to test it in more complicated systems for fear that the tests would not support the model. But the tests *did* support the model, and thereafter the

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Events Calendar

More information about the following events will appear at our website at `http://www.math.usf.edu` as the event approaches. So mark the event on your calendar, and plan on taking part.

Nov. 12, 1999: **Mathematics Field Day.** We will be having a Field Day on Friday, Nov. 12. We invite high schools in Hillsborough, Manatee, Pasco, Pinellas, and Sarasota counties to send math teachers and high ability juniors to USF-Tampa for a day of mathematics. See the CMS news section for details. For information about the CMS, consult our website or call (813) 974-4068.

Dec. 3, 1999: **MAA Suncoast Meeting.** The 24th annual Suncoast Regional Meeting of the Florida Section of the MAA will be held at USF-Tampa on Dec. 3, starting 3 pm. This event brings together teachers of mathematics from middle to graduate features an afternoon of mathematics talks, followed by dinner at the Marshall Center. For information on registration and/or giving talks, contact Fred Zerla at `zerlamath.usf.edu`.

Mar. 3 & 4, 2000: **MAA Florida Meeting.** We will host the 33d Annual Meeting of the Florida Section of the MAA on March 3 & 4, 2000, in the Business Building. The Local Arrangements Committee consists of G. McColm, K. Pothoven and F. Zerla. The last time USF hosted this meeting was in 1977, and we would appreciate your help, suggestions and participation to make this once-in-a-quarter-century event memorable.

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development of the theory of infinite dimensional matrices ("operators"), driven by the theory of "function spaces" in mathematics went hand in hand with the development of quantum physics.

There are many applications of pure mathematics to the outside world. Some well known examples are: mathematical logic in the development of computers; stochastic processes in stock market predictions; prime factorization/number theory in secret codes (both military and public use); and harmonic analysis in signal processing and picture compression/recognition.

Some Clouds

Recently we have encountered a similar coincidence in our research at USF, in "approximation theory". (In our department there is a strong group working in approximation theory and related areas.) The main goal of *approximation theory* is to approximate complicated objects (like functions, surfaces, sets, etc.) by simpler ones (like polynomials, polygonal surfaces, level sets etc.). The idea is to select simpler objects "close to" the complicated one which preserve the properties of the latter. One would expect that the simple object, which could be handled more easily, could be studied in place of the more complicated one in most relevant questions.

For example, suppose that we want to transmit a sound file. A sound can be regarded as an infinite sum of pure tones (sine waves), most of them inaudible. We could just use the finite sum of audible tones, and a listener would not be able to tell the difference: the finite sum would be a *good approximation* of the original sound. And instead of transmitting the original sound, all we have to forward is the size (= coefficients) of those audible pure tones (sine waves), and the reconstruction of the approximation can be easily done by the receeiver. This is a much less demanding task than sending the whole sound spectrum for the original sound.

Unfortunately, nothing is free in life, and in general we pay the price for simplicity by having poor approximations. A very elaborate theory has evolved since the second part of the last century that deals with the relationship of simplicity versus good approximation. One of the basic approaches is to have a collection of nice things — e.g., pure tones — so that we could have a finite sum of these to approximate a less nice thing — e.g., a sound. One standard tool is the use of a collection of "orthogonal polynomials" to approximate functions: a function could be approximated by a finite sum of these "orthogonal polynomials".

It is often said that any formula in an expository article cuts readership in half, so to avoid the risk, we omit the definition of "orthogonal polynomials." The idea is that a collection of polynomials are "orthogonal" if they are (in a sense) mutually perpendicular with respect to a weight function. (Technically, the weight function is a function w, and two polynomials f and g are orthogonal with respect to w if the definite integral of the product fgw is 0.)

One of the most classical examples of a class of orthogonal polynomials are the "Hermite polynomials" that are orthogonal with respect to the weight function $w(x) = e^{-x^2}$. The theory for Hermite polynomials is well developed and understood. But if we used the weight functions $w(x) = e^{-x^a}$ with some $a \neq 2$, then the behavior of these polynomials is much more difficult to describe. Major contributors to the the-

ory of these more difficult weight functions are among our faculty: E. A. Rahmanov, E. B. Saff and recently M. H. Ismail. The major insight came from applying some purely mathematical methods with a root in physics: potentials in the presence of an "external field".

Approximation of Clouds

Over the last decade or so it has turned out that this more difficult theory has numerous applications in a variety of mathematical problems. E. B. Saff and I have been preparing a monograph on the subject (*Logarithmic Potentials with External Fields*) that was to be published in Springer's *Grundlehren der mathematischen Wissenschaften* series, which is probably the most prestigious old series of mathematical texts. One chapter of the book was devoted to the aforementioned orthogonal polynomials. Just before completing the manuscript, we learned that some theoretical physicists had been dealing with the same problem: describing the behavior of these more difficult orthogonal polynomials. They were working on statistical-mechanical models of quantum systems. They replaced quantum particles with mechanical balls, but to account for the quantum behavior they used a random distribution W of these balls and W could be a weight. Then orthogonal polynomials with respect to W emerge in the calculation of physical quantities like the density of states.

In science one often encounters "similar" questions, that, under closer scrutiny, turn out to be totally unrelated to what one is doing. However, our case was a perfect match, but the method of the physicists was completely different: they were using random matrix theory. We have been quite surprised by this unexpected coincidence, for our investigations were motivated by purely theoretical questions in approximation theory with no reference to physical applications whatsoever. As a result, we have become richer by an application of our methods, and at the same time provided a rigorous framework for proving results concerning random matrix models.

So after all is all mathematics relevant to the real world or should it be? This may be a question we should leave for philosophers to answer; but it is also a question that is debated in various forms ("good versus bad" mathematics; "pure versus applied" mathematics) over and over again. The anwer is probably "no", but in any case one should exercise some caution in harsh judgements: remember that one of the pearls of pure mathematics, the "little" Fermat theorem, had been sitting there as a beautiful but useless porcelain for over three hundred years before, in the late 1970's, it became the basis of public key cryptography and today the security of millions of bank transactions depend on it.

Let us finish with a quote from Eugene P. Wigner, who was a Nobel laureate physicist and at the same time an outstanding mathematician. "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research, and it will extend ... to wide branches of learning."

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Mar. 9, 2000: **Nagle Lecture.** The Nagle Lecture Series has invited Jerrold Marsden to give the Nagle Lecture on natural and artificial satellites on Thursday, March 9, at 7:30 pm, place TBA.

Faculty News

Last Spring, *Mourad Ismail* spent a semester at the Mathematical Science Research Institute at UC Berkeley after being invited to their half-year program on Random Matrics. He was also awarded a grant from the National Science Foundation, for 1999-2002, for his work in Orthogonal Polynomials and Special Functions, for $ 123,000.

Manoug Manougian won a Teaching Incentive Program award in the 1998-99 year. The TIP program was established by the Legislature to encourage high-quality teaching.

For many years, *Fred Zerla* served as our Undergraduate Advisor. The Undergraduate Advisor is the one who figures out how undergraduates can navigate through the bureaucracy to get the academic program that they want. He has passed the sword and shield of office to *Mile Krajcevski*, who starts out this year as our new Undergraduate Advisor.

Ed Saff will be the editor of the new journal *Foundations of Computational Mathematics*, whose first issue will come out in January, 2001. He was also named *Erskine Fellow* at the University of Canterbury, in New Zealand.

We have *six* visitors this year. Arthur Danielyan and Serguei Piskarev are visiting from Russia. Gang Joon Yoong is visiting from South Korea. Vasyl Kurta is visiting from the Ukraine. Jose Antonio Gerra and Luz Marina Suarez are visiting from Venezuela.

CMS News

The Center for Mathematical Services continues to be involved in oureach and service activities to the area counties served by the University of South Florida.

This past Summer the Center conducted three programs for gifted and high-ability secondary students. All programs ran concurrently from June 10 through July 9 from 9 am to 4 pm daily. This was the twenty-first year that the Center has conducted such programs. The Mathematics and Engineering Program had 45 high school students from Hernando, Hillsborough, and Pasco counties. The Biomedical and Life Science Program had 35 students from Hillsborough, Manatee, Pasco, and Sarasota counties. The Mathematics Program had 110 students from Hernando, Hillsborough, and Pasco counties. All the students were taught in formal class settings but also participated extensively in appropriate laboratory exercises in science and computer labs. The Center is planning on conducting similar programs during the Summer of 2000.

On November 12, the Center is hosting the third annual Mathematics Field Day. We expect 150 high school juniors (from high schools in Hillsborough, Manatee, Pasco, Pinellas, and Sarasota counties — at most four or five per school) and forty mathematics teachers. They will be treated to talks by Professor Oberste-Vorth on *Fractal Geometry* and by Professor Boris Shekhtman on *Life in Many Dimensions*. In addition, new to this year's event will be a mathematics competition where teams from the various high schools will be competing against each other. The students will aslo be given a tour of the campus during their stay by undergraduate University ambassadors while the teachers will be treated to a luncheon in the Marshall Center and an address by the Undergraduate Dean Robert Sullins.

Information about activities of the Center can be obtained by calling (813) 974-4068.

Student News

Louis Camara, President of the USF Chapter of Pi Mu Epsilon (the mathematics honor fraternity ΠΜΕ), swept the undergraduate honors in the Department this year with the $ 500 Mathematics Department Scholarship in January, first prize for student presentations Student Conference in the Mathematics Association of America Florida Section Meeting in March, and being selected the *Outstanding Scholar of the Florida Epsilon Chapter of ΠΜΕ for 1999* in April, the latter an award bestowed on the top senior mathematics student at the University. Louis will present his paper on "The General Term of the Pell Sequence" at the ΠΜΕ Session of *Mathfest99*, in Providence, Rhode Island. A native of Senegal, Louis is a BA/MA student in Mathematics, and he plans to continue his study of Mathematics at the doctoral level here at USF.

We congratulate the students who received the Bachelor's Degree in Mathematics this past year. In Fall, 1998: Jamie Alexander Battles, Constantine Chris Chamis, Wakako Ihara, Michael William

Santore (Magna Cum Laude), Angela A. Smith, and Michael S. Ashton (INM). In Spring, 1999: Melinda Kaye Bentley, Ryan Bernard Bethune, Delvin Charles Genenbacher, William Henry Hughes, Angela Marie Owens, and David Herbert Perlow. In Summer, 1999: Patrick Devereux Kile, Elesea Eljaquitta Page, Glen David Parker, Beverly Anne Sasser (Summa Cum Laude), Marnelle E. Valcin, and André Cedrick Williams.

We congratulate the students who received the Master's Degree in Mathematics in Spring, 1999: Christopher Brown, Ying Chen, Ming Dai, Abdeljalil Douiki, William Henry Hughes, Kathy Lora Jensen, Jost Heinrich Thias, Francisco Javier Traverso-Garcia, Lixin Wang, and Lei Zhang.

Student Clubs

The ΠME and MAA Math Clubs began the Spring Semester with a presentation by representatives of the U.S. Air Force on mathematics opportunities in the Air Force. Three members of the Mathematics Faculty spoke at meetings: Masahico Saito discussed "Knot Colorings and Algebraic Structures," Boris Shekhtman talked about "Amazing Problems in Interpolation," and Ralph Oberste-Vorth gave insights on "Newton's Methods". William Hughes, candidate for the Bachelor/Masters in Mathematics discussed "Malicious Code: Melissa and Others". The Math Clubs hosted two Hillsborough County Math Bowls this year. The Fall Math Bowl in December was attended by approximately 250 high school students and teachers while the Spring Math Bowl in April had almost 300 attendees. At the ΠME Induction Banquet, seven new members were inducted to the honor society, and I. A. Sakmar, Visiting Professor of Physics, told of the interplay between mathematics and physics.

Opportunities

Fellowships for Graduate Students. The Department of Mathematics offers several forms of financial support to qualified graduate students

We offer teaching assistantships, which start at $ 11,000 (and include a waiver of about 80 % of the tuition), and go higher for students in the Ph.D. program. A teaching assistant is required to sign up for 9 credit hours each semester and stay in good academic standing.

The teaching assistantships or fellowships can be increased up to $ 15,000 by the funds provided from the Tharpe foundation. Each year we have an opportunity to award a few of our exceptional students with these additional funds.

Beginning August, 2000, new students may compete for a University Graduate Fellowship of at least $ 11,000. This fellowship does not require students to do any additional work, and also brings a tuition waver of approximately 80 %.

Instructorships. The Mathematics Department will be conducting a hiring search to hire three instructors to teach lower-division service courses (like College Algebra, Finite Mathematics, Elementary Calculus, etc.). The duties will consist entirely of teaching these courses, assisting in the management and oversight of such courses, and various administrative duties. We are hoping to hire people with considerable teaching ability and commitment; a doctorate is required. For further information, see our website, or contact us at `mathsearch@math.usf.edu` or (813) 974-2643.

Feedback

We always like feedback: we are trying to reach out to the community. Our website is at:

`http://www.math.usf.edu`

We can be reached by e-mail at:

`mathdept@math.usf.edu`

Our phone number is (813) 974-2643; our fax number is (813) 974-2700; and our mailing address is:

Department of Mathematics
University of South Florida
4202 E. Fowler Ave., PHY 114
Tampa, FL 33620